Statistical Approaches to the Even-odd Effect in Fission

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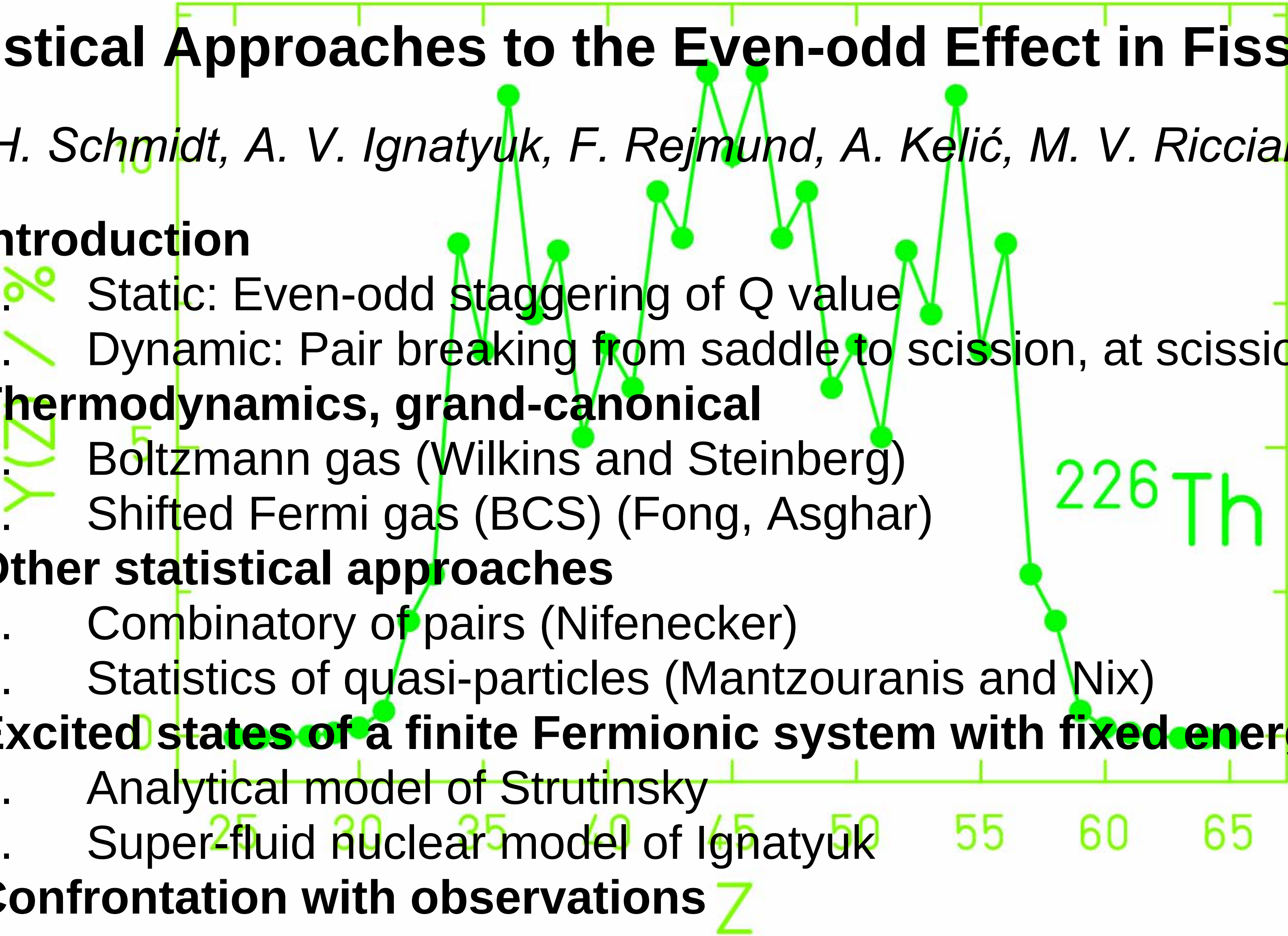
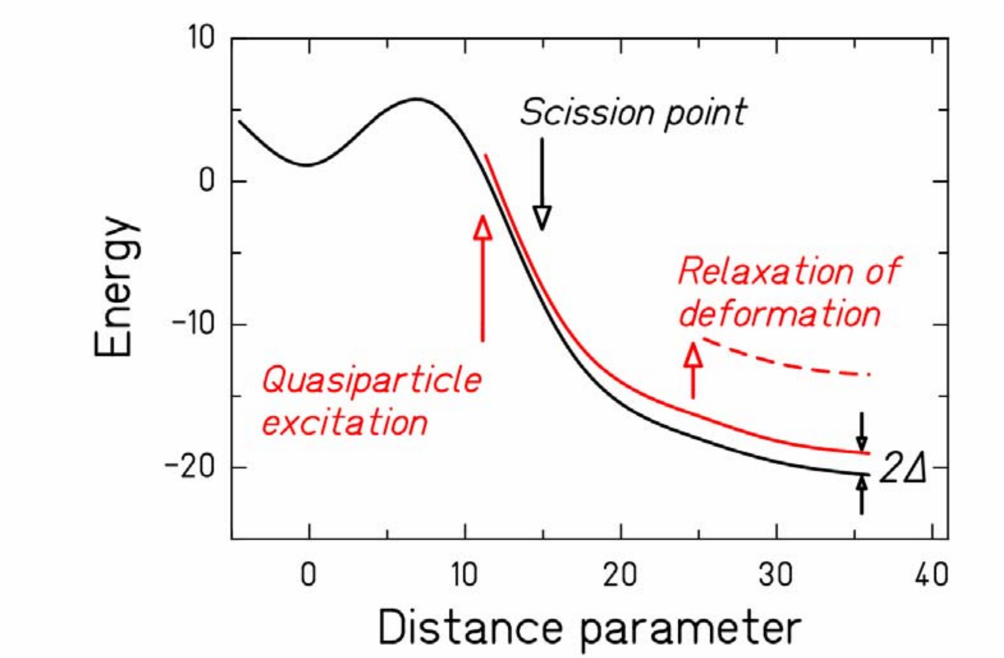
1. **Introduction**
   a. Static: Even-odd staggering of Q value
   b. Dynamic: Pair breaking from saddle to scission, at scission
2. **Thermodynamics, grand-canonical**
   a. Boltzmann gas (Wilkins and Steinberg)
   b. Shifted Fermi gas (BCS) (Fong, Asghar)
3. **Other statistical approaches**
   a. Combinatory of pairs (Nifenecker)
   b. Statistics of quasi-particles (Mantzouranis and Nix)
4. **Excited states of a finite Fermionic system with fixed energy**
   a. Analytical model of Strutinsky
   b. Super-fluid nuclear model of Ignatyuk
5. **Confrontation with observations**



Static and dynamical aspects of the even-odd effect in fission



Case: Even-Z nucleus passes the fission barrier fully paired.

Even and odd charge splits differ in Q value by Δ.

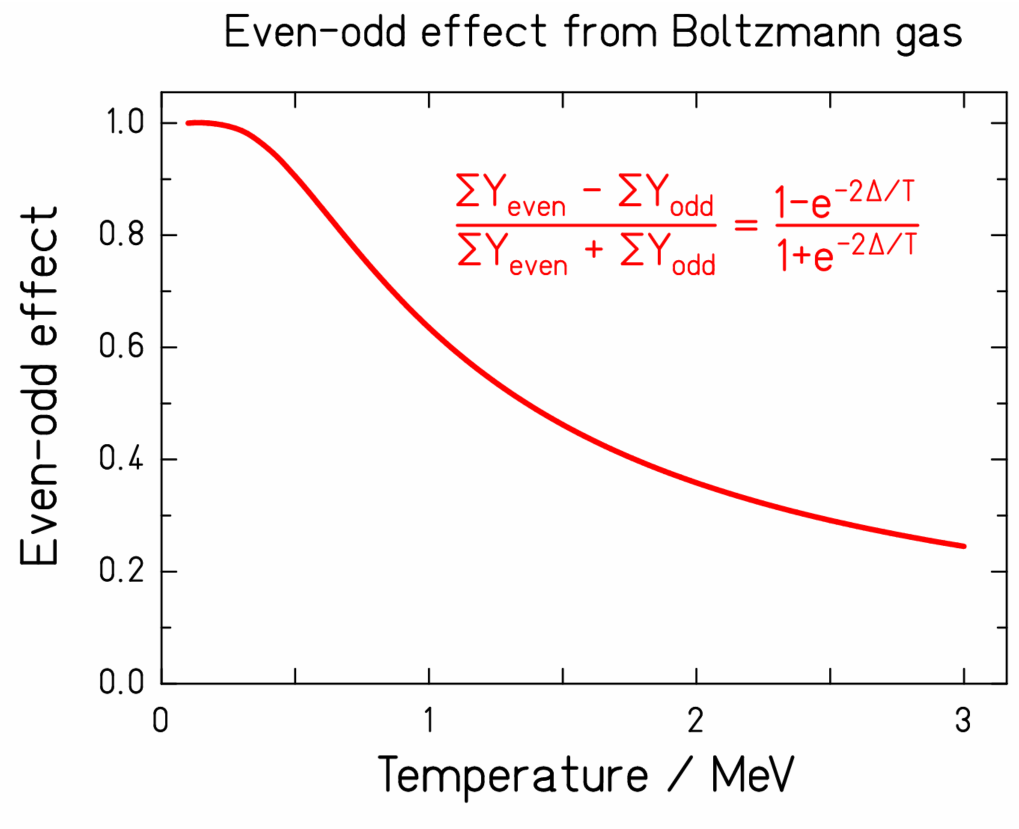
Odd-Z fragments can only be produced by quasi-particle excitations before or at scission.

Controversy:
Can the statistical model "explain" the even-odd effect in the yields?

- **Quasi-particle excitations on the way from saddle to scission.**
- **Yields of different fragments due to the number of available states.**

Alternative option: Pair breaking during neck rupture.

Thermodynamical approach



Ideal gas:

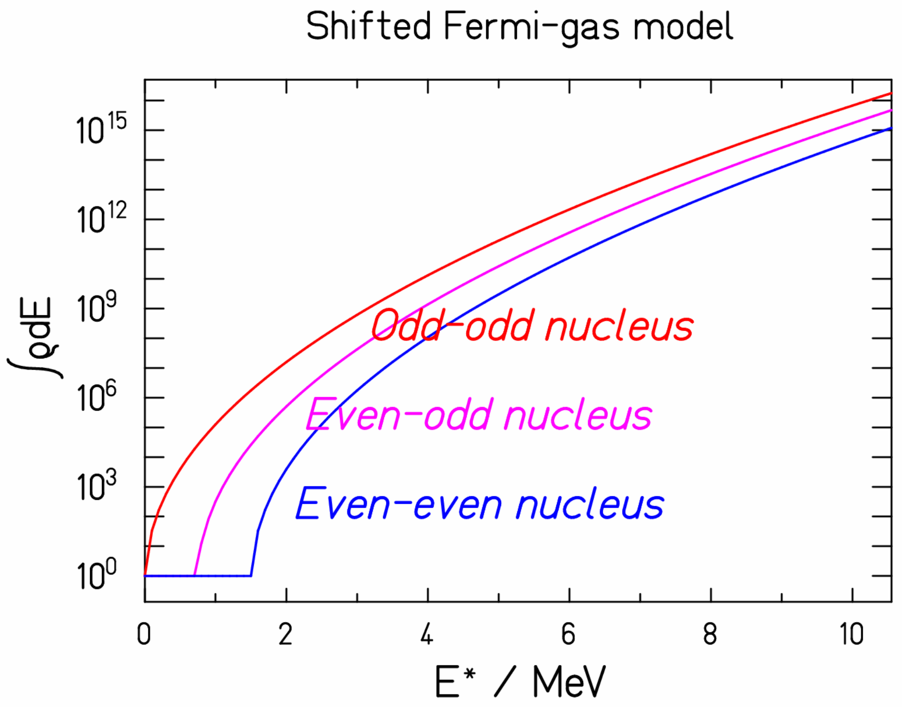
$$y \propto \exp\left(-\frac{E}{T}\right)$$

(Boltzmann)

Applied e.g. by Wilkins and Steinberg, PLB42 (1972) 141, in their scission-point model.

Not appropriate for finite super-fluid Fermionic system!

Shifted Fermi-gas model



Boltzmann replaced by Fermi-gas level density

Asymptotic behaviour modelled by shift of
Δ for even-odd
and
2 Δ for even-even nuclei.

Even-odd fluctuations in Q value are exactly balanced by shift of level density (e.g. Medkour et al., JPG 23 (1997) 103).
No even-odd effect in yields expected!

Other statistical approaches

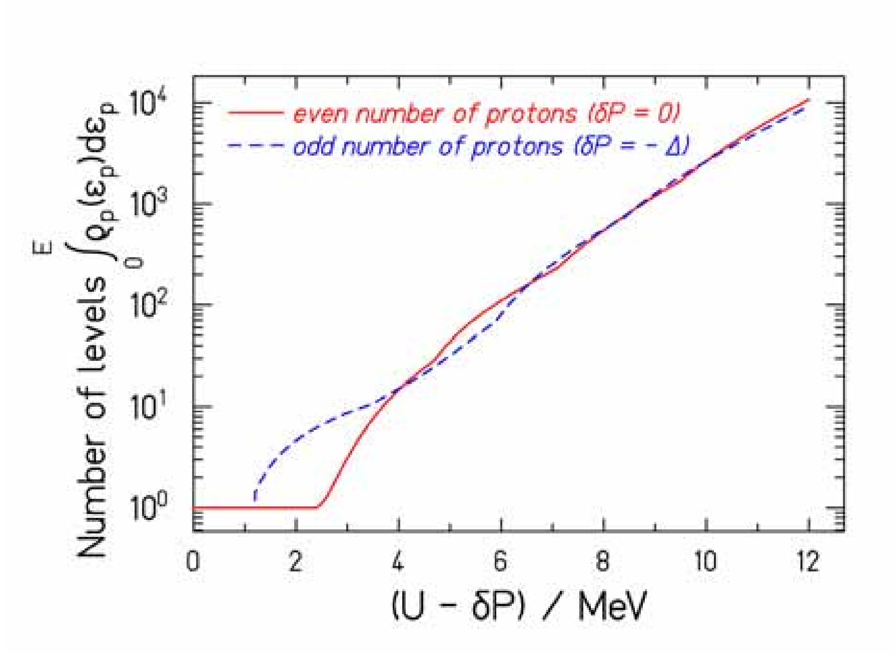
Nifenecker et al. ZPA308 (1982) 39:
Statistical combinatorial approach based on the number of broken pairs.

Mantzouranis and Nix, PRC25 (1982) 918:
Statistical approach based on the ratio of the number of quasiparticle excitations.

Both approaches are not consistent with the basics of statistical model:
The number of available states.

Excited states of a super-fluid Fermionic system

1. Analytical approximation of Strutinsky (1958)



- **Fixed energy (not temperature!)**
- **Super-fluid system**
- **Restriction to one component**

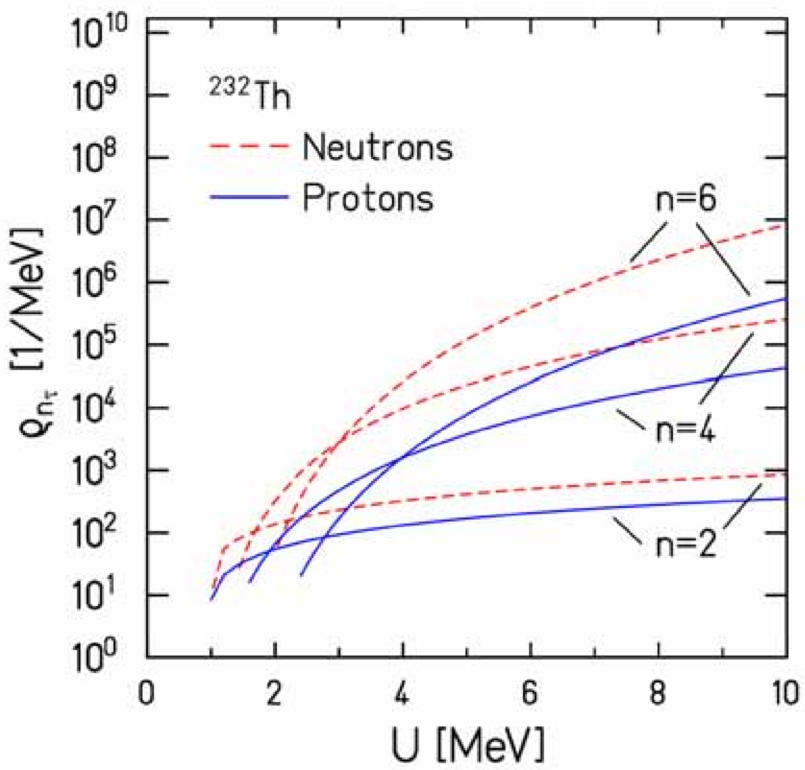
$$\rho_n(U) = \frac{g^n (U - n\Delta)^{n-1}}{\left[(n/2)!\right]^2 (n-1)!}$$

Approximation: **Δ does not depend on energy. (This is not critical for the lowest excitation energies considered here)**

Relation to shifted Fermi-gas model:

1. **Deviations for lowest energies appear!**
2. **Extension to two-component system required!**

Excited states of a super-fluid Fermionic system



2. Analytical description of Ignatyuk et al. SJNP17 (1973) 376

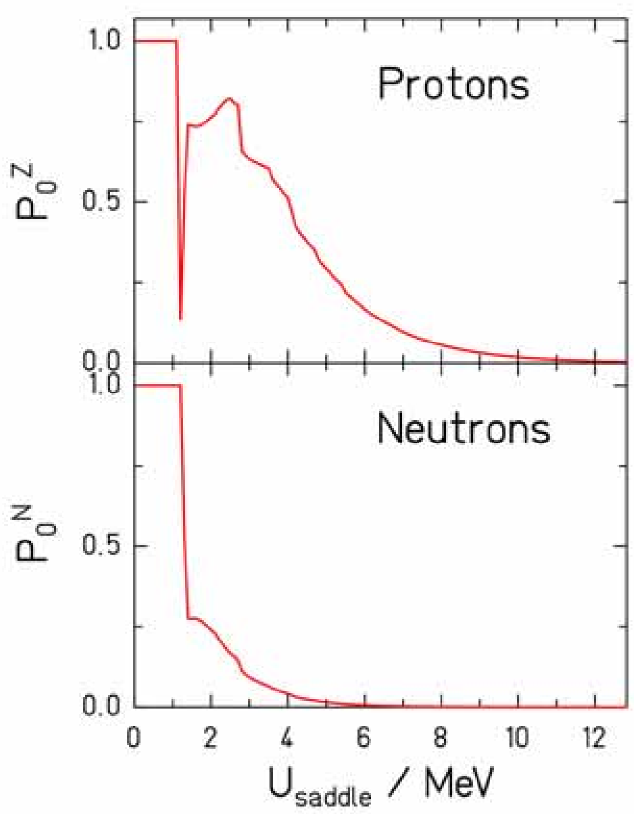
- **Extension to two-component system**
- **Inclusion of Pauli blocking**
- **Variation of pairing strength**

Stringent formulation of the number of excited states in the proton and neutron subsystem with N quasiparticles.

Neutron and proton excitations with different numbers of quasiparticles are in competition!

Excited states of a super-fluid Fermionic system

3. Application to fission by Rejmund et al. NPA678 (2000) 215



Probability for fully paired proton configuration:

$$P_0^Z(U) = \sum_{n_N} \rho_{n_Z=0,n_N}(U) \Big/ \sum_{n_Z,n_N} \rho_{n_Z,n_N}(U)$$

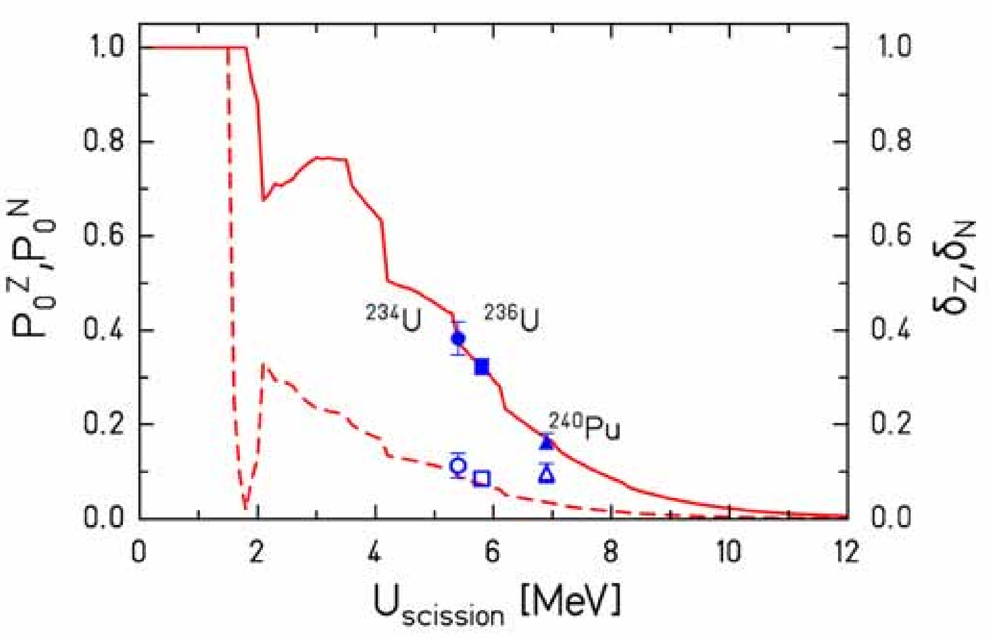
(P_0^Z = observed even-odd effect!)

Finite probability for U > Δ that protons (neutrons) remain completely paired.

Purely statistical reasoning can explain the even-odd effect in fission!

Observations

1. **Magnitude of proton and neutron even-odd effects,**
2. **Energy dependence**

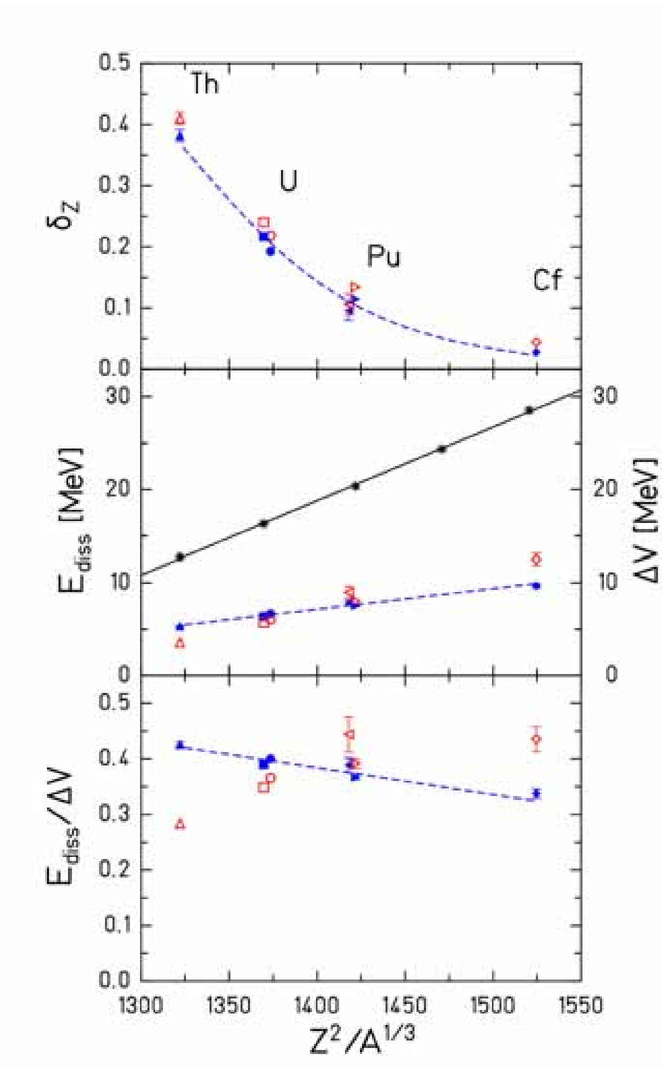


Explanation of drastic difference between proton and neutron even-odd effect.

"Reasonable" excitation-energies at scission.

Data for high TKE: no neutron evaporation possible.

Deduced intrinsic excitation energy at scission



δ_Z: Fraction of proton QP excitations at scission

Rejmund et al. NPA678 (2000) 215

Nifenecker et al. ZPA308 (1982) 39

ΔV: Potential energy gain (saddle-scission)

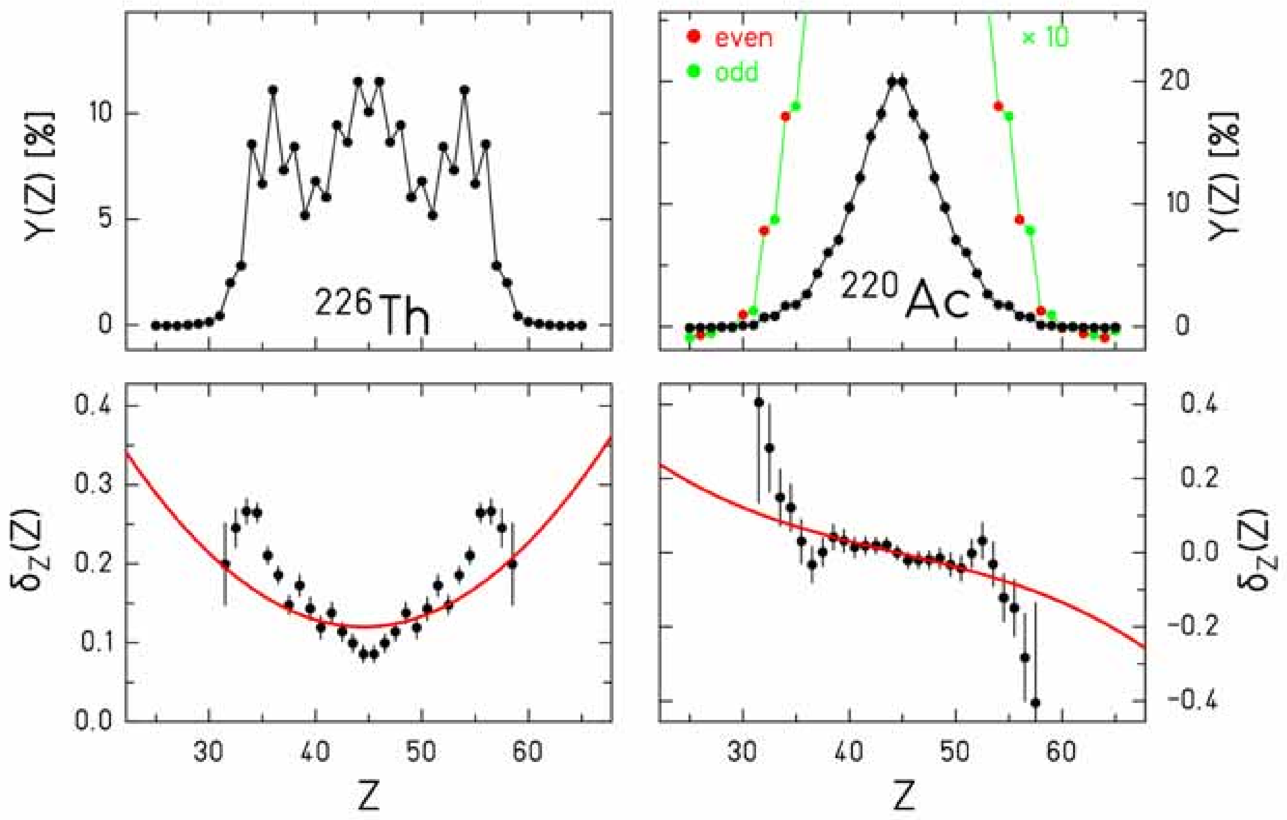
M. Asghar, R. W. Hasse, JP **C6** (1984) 455

E_{diss}: Deduced dissipated energy

$E_{diss}/\Delta V$: 30% to 40%

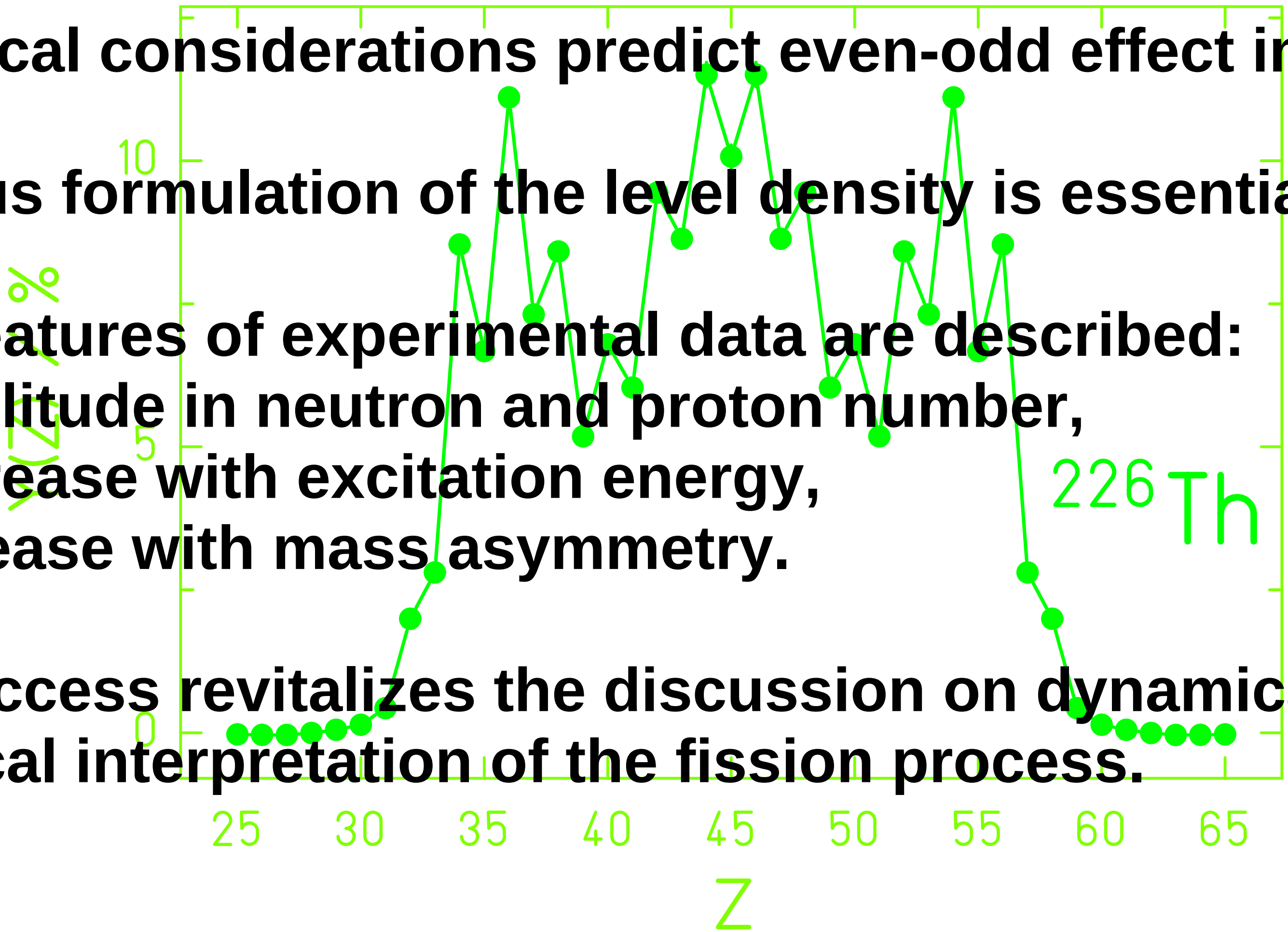
Observations

3. Variation with asymmetry (Steinhäuser et al. NPA634 (1998) 89)



Even-odd effect in mass-asymmetric splits (also for odd-Z nuclei!) due to larger single-particle level density in larger fragment.

Conclusion



Statistical considerations predict even-odd effect in fission.

Rigorous formulation of the level density is essential.

Many features of experimental data are described:

- **Amplitude in neutron and proton number,**
- **Decrease with excitation energy,**
- **Increase with mass asymmetry.**

This success revitalizes the discussion on dynamical or statistical interpretation of the fission process.