The fission rate in multi-dimensional Langevin calculations

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Motivation

Understanding of collective properties of nuclei

Large difference in the predictions of dissipation properties in different theoretical models

Experimental data on nuclear dissipation have often been interpreted using one-dimensional model calculations of the Langevin or Fokker-Planck type.

Investigate the influence of the dimensionality of the deformation space on the fission process in Langevin calculations.

Theoretical description of fission

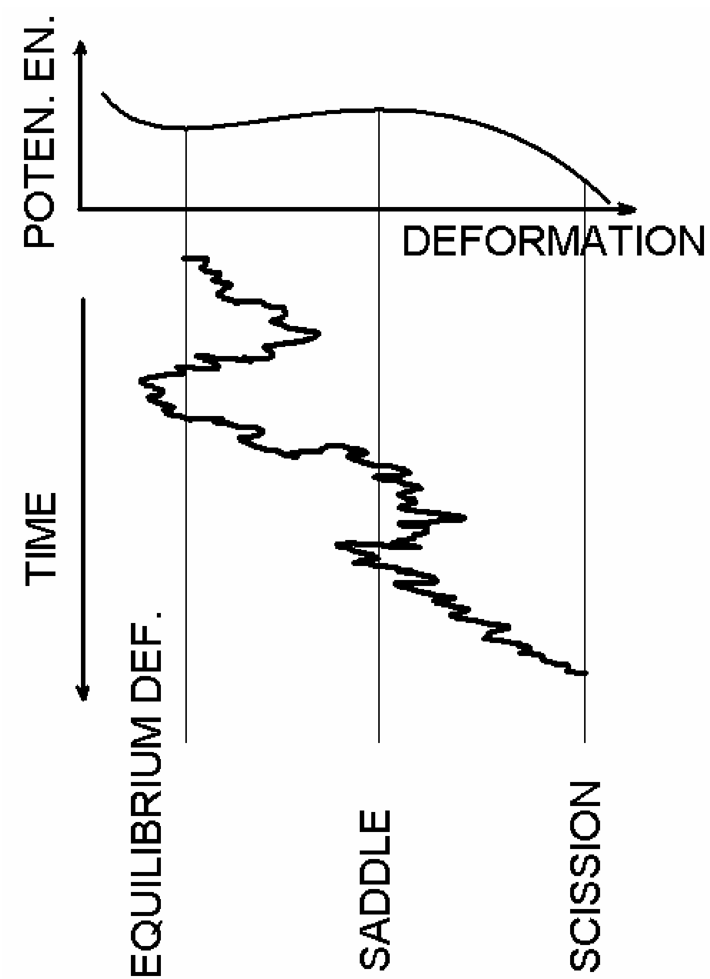
stochastic approach

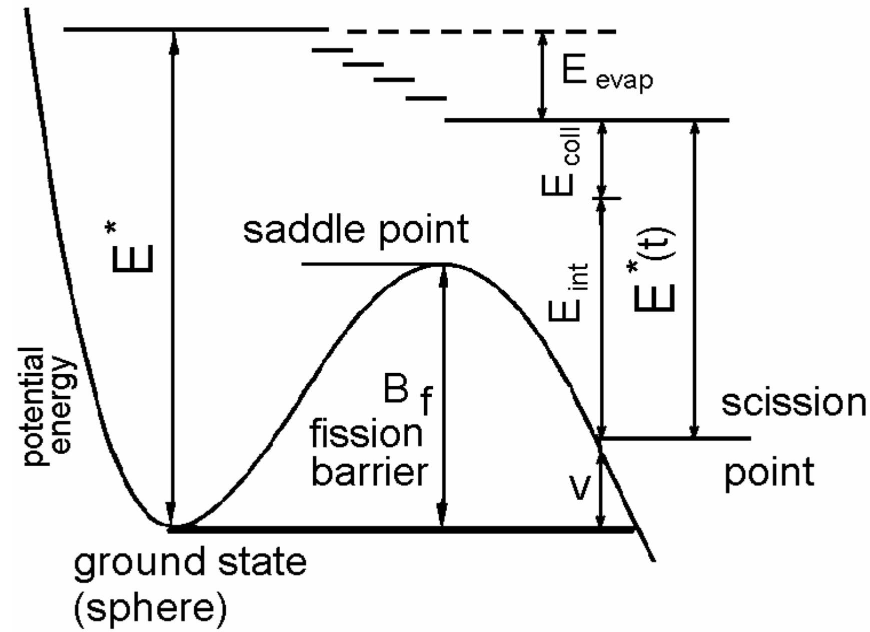
collective variables (shape of the nucleus)	internal degrees of freedom (“heat bath”)

Langevin equations

Langevin equations describe time evolution of the collective variables like the evolution of Brownian particle that interacts stochastically with a “heat bath”.

The stochastic approach





E_{coll} - the energy connected with collective degrees of freedom

E_{int} - the energy connected with internal degrees of freedom

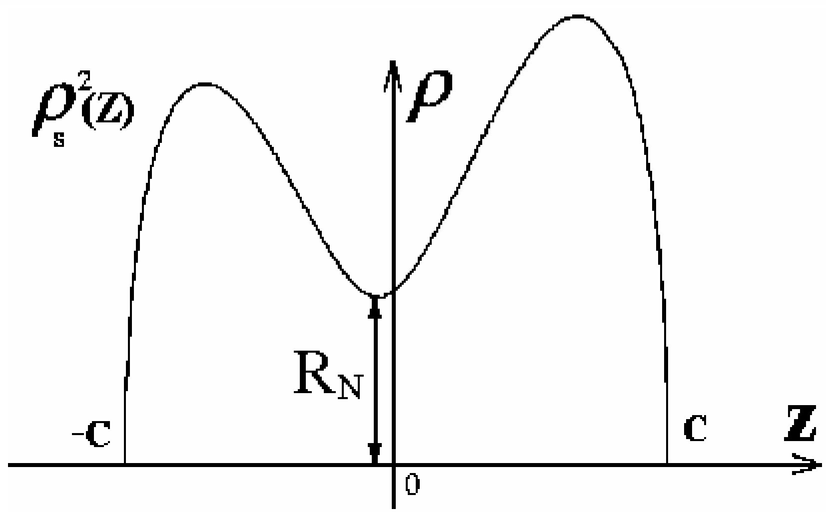
E_{evap}- the energy carried away by the evaporated particles

The (c,h,a)-parameterization

$$\rho_s^2(z) = \begin{cases} \left(c^2 - z^2\right)\left(A_s + Bz^2/c^2 + \alpha z/c\right), & \text{if} \quad B \geq 0; \\ \left(c^2 - z^2\right)\left(A_s + \alpha z/c\right)\exp\left(Bcz^2\right), & \text{if} \quad B < 0, \end{cases}$$

$$B = 2h + \frac{c-1}{2}.$$

$$A_s = \begin{cases} c^{-3} - \frac{B}{5}, & \text{if} \quad B \geq 0; \\ -\frac{4}{3}\frac{B}{\exp(Bc^3) + \left(1 + \frac{1}{2Bc^3}\right)\sqrt{-\pi Bc^3}\,\mathrm{erf}\left(\sqrt{-Bc^3}\right)}, & \text{if} \quad B < 0. \end{cases}$$



The Langevin equations

$$\dot{q}_i = \mu_{ij} p_j,$$

$$\dot{p}_i = -\frac{1}{2} p_i p_k \frac{\partial \mu_{jk}}{\partial q_i} - \frac{\partial F}{\partial q_i} - \gamma_{ij} \mu_{jk} p_k + \theta_{ij} \xi_j (t)$$

q - collective coordinate **p** - conjugate momentum

$\mathrm{m}_{ij}(\|\mu_{ij}\| = \|\mathrm{m}_{ij}\|^{-1})$ - inertia tensor

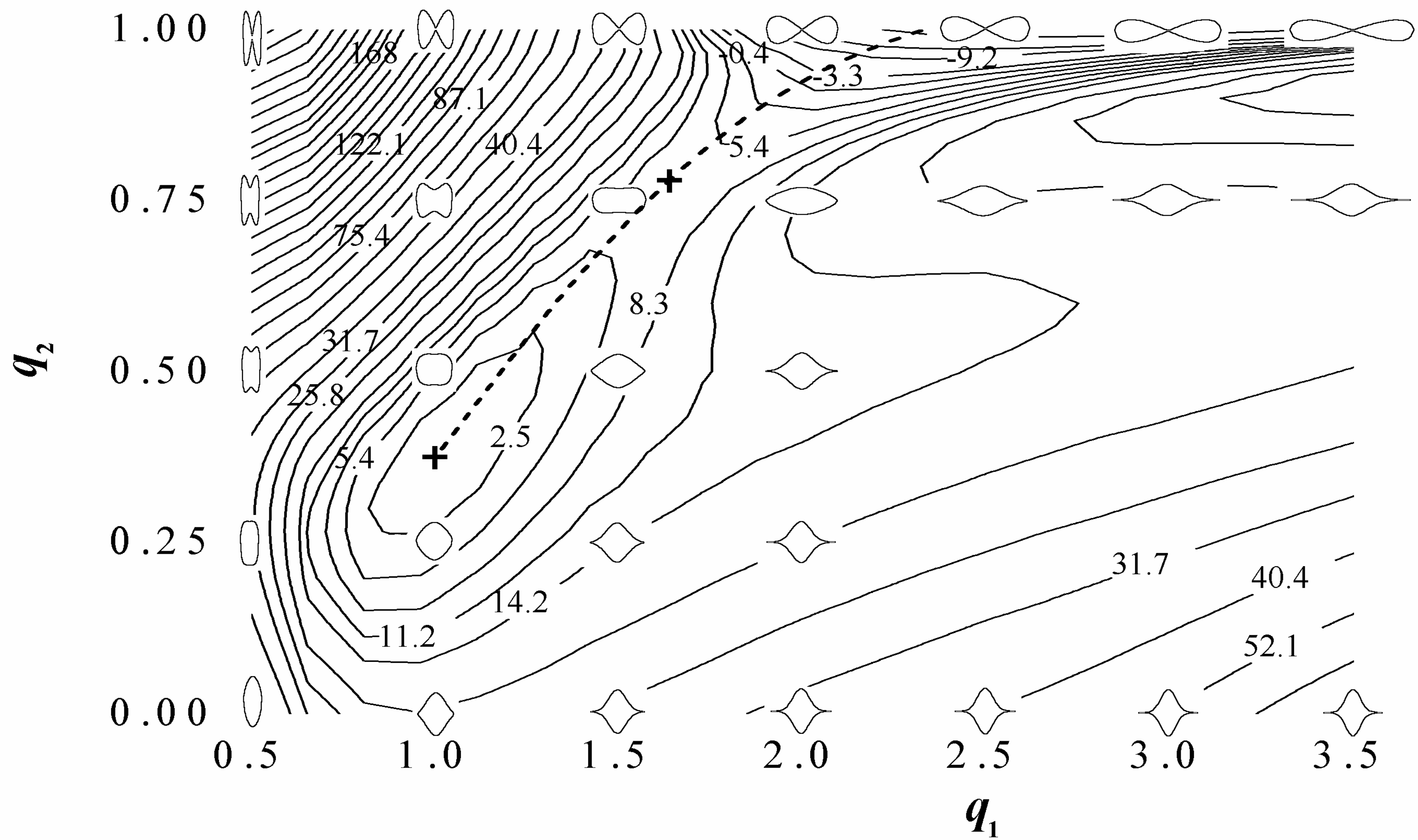
$F(\mathbf{q}) = V(\mathbf{q}) - a(\mathbf{q})T^2$ - Helmholtz free energy

$a(\mathbf{q})$ - level density T - temperature

V - potential energy γ_{ij} -friction tensor

$\theta_{ij}\xi_j$ - random force ξ_j - random variable

The collective coordinates



$q_1 = c;$

$q_2 = (h+1.5)/(h_{sc}+1.5);$

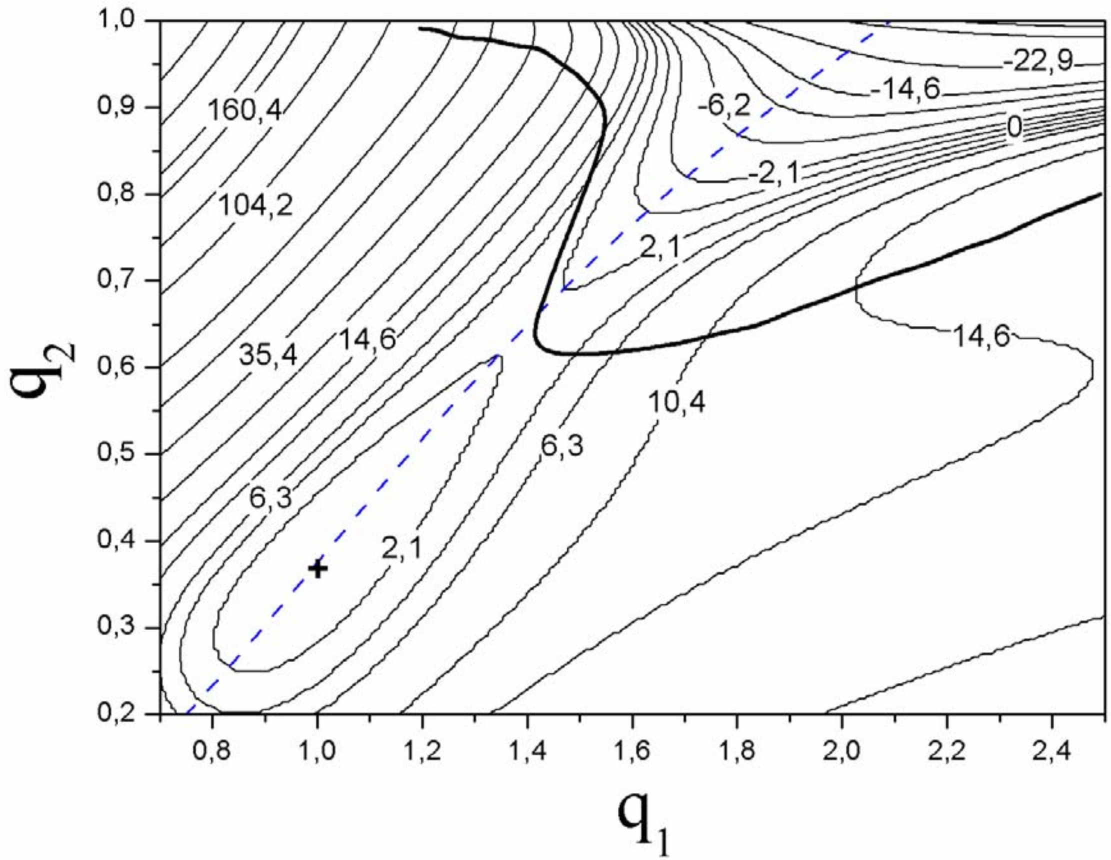
$h_{sc} = 5/(2c^3)+(1-c)/4;$

$q_3 = \alpha/(As+B)$ if $B \geq 0$

$q_3 = \alpha/As$ if $B<0$

Nadtochy and Adeev PRC, 2005

The collective coordinates



The potential energy for the ^{248}Cf.

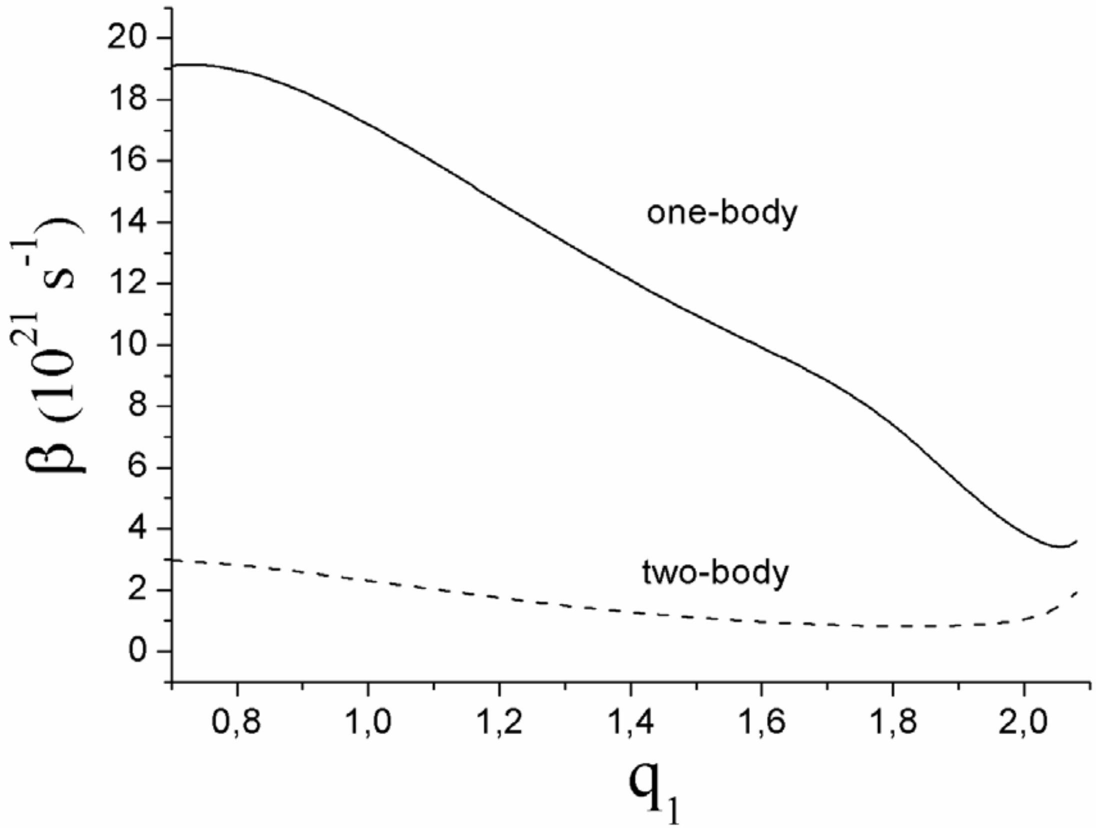
The solid curve – saddle point configurations.

The dashed curve – fission valley.

Calculations:

1. One – dimensional (bottom of fission valley) h=a=0
2. Two – dimensional (symmetrical fission) q_3=0, **q**=(q1,q2)
3. Three – dimensional **q**=(q_1,q_2,q_3).

The friction parameter



The friction parameter

$$\beta = \gamma / m$$

- One-body dissipation
  ("wall" and "wall-plus-window" formulas)
- Two-body dissipation with the two-body friction constant
  $\nu_0 = 2 \text{ x } 10^{-23}$ MeV s fm^{-3}

Results of calculations

1-, 2-, and 3-dimensional

calculations for

^{248}Cf at $E^* = 30$ MeV and $E^* = 150$ MeV;

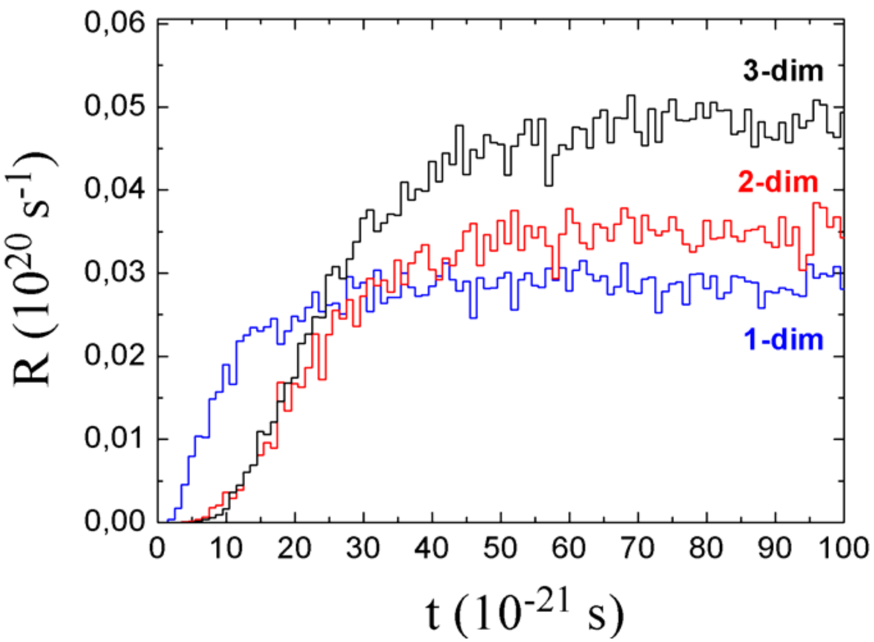
^{213}At at $E^* = 190$ MeV

$$\text{Fission rate: } R(t) = -1/N(t)\, dN(t)/dt$$

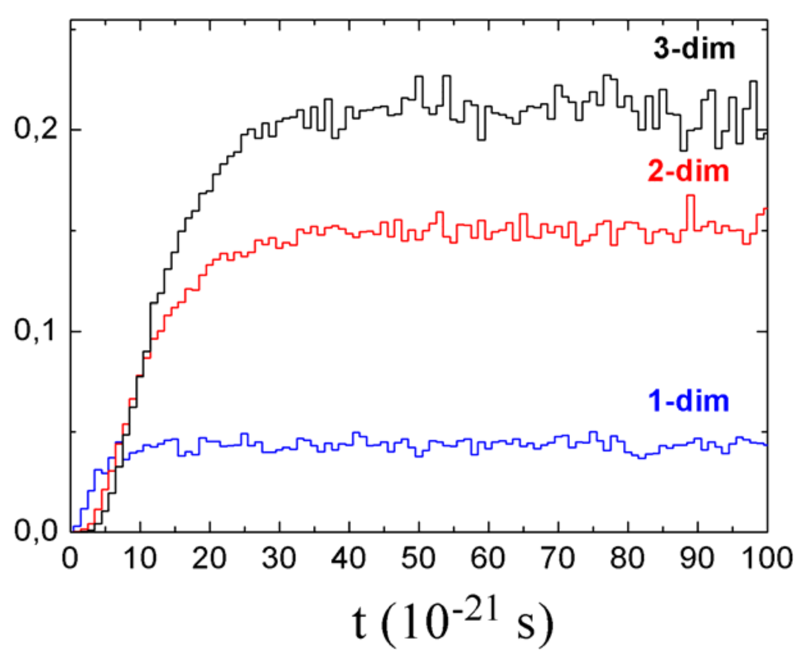
N(t) - the number of Langevin trajectories, which did not escape beyond the saddle at time t.

The results for the ^{248}Cf (one-body)

E*=30 MeV



E*=150 MeV



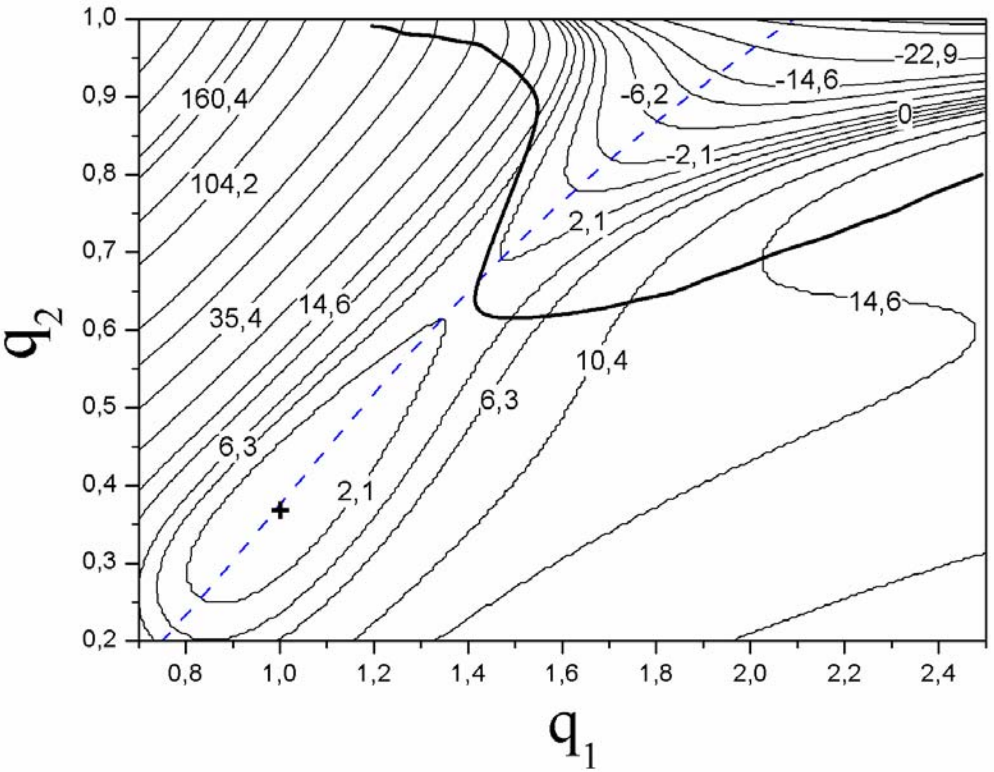
The stationary values:

$R^{3d}/R^{1d} = 1.8$

$R^{3d}/R^{1d} = 5$

The lowest transient time – one-dimensional calculations

The results for the ^{248}Cf

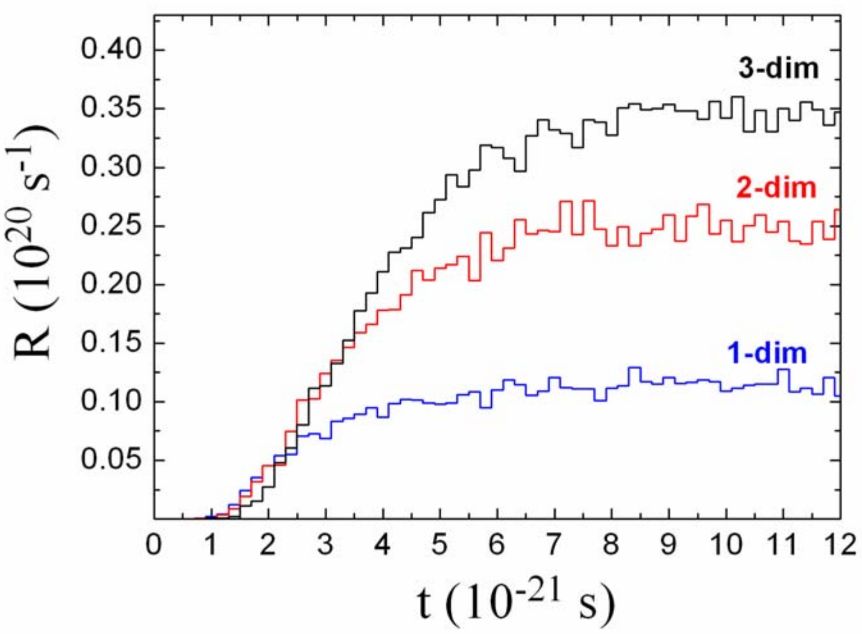


one-dimensional case:
R(t) - flux over only saddle point

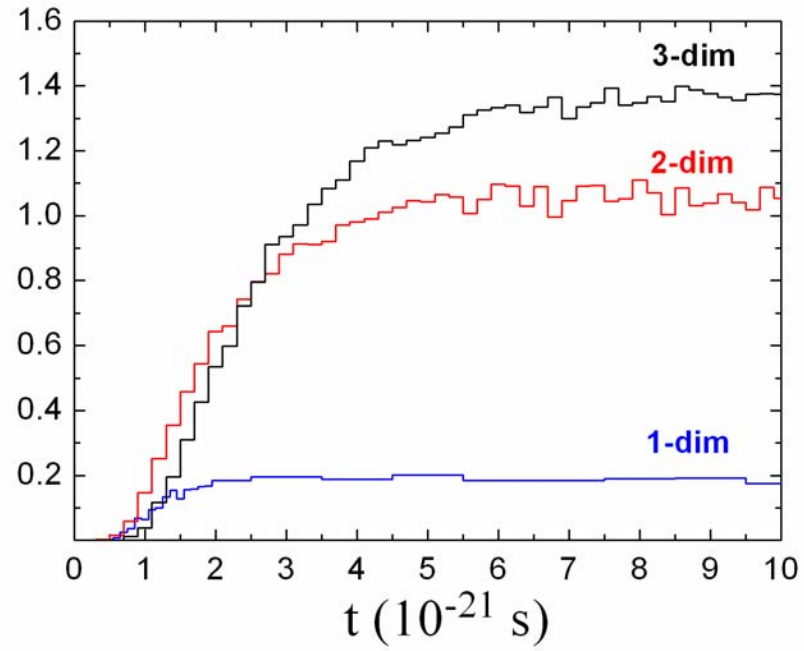
multi-dimensional case:
R(t) - flux over populated saddle point configurations

The results for the ^{248}Cf (two-body)

E*=30 MeV



E*=150 MeV



The stationary values:

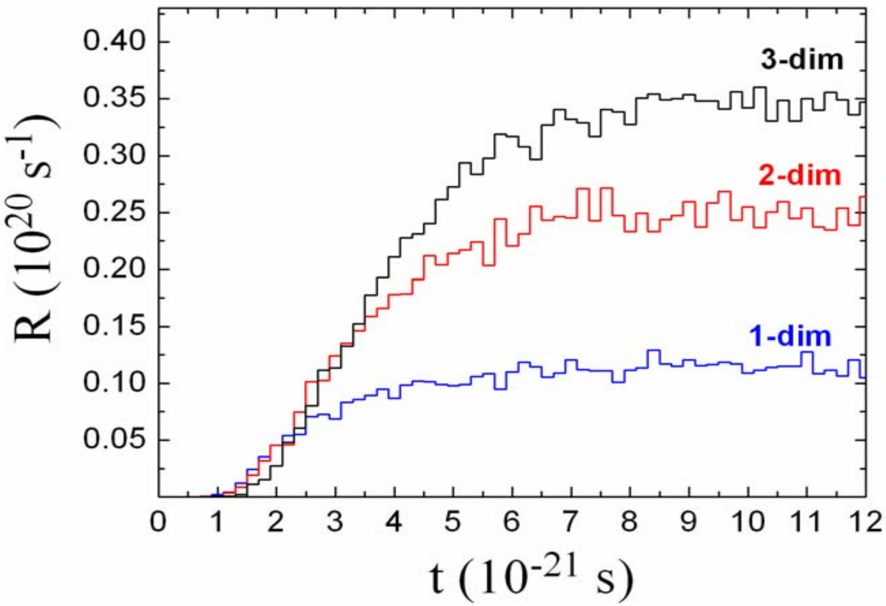
$R^{3d}/R^{1d} = 3$

$R^{3d}/R^{1d} = 8$

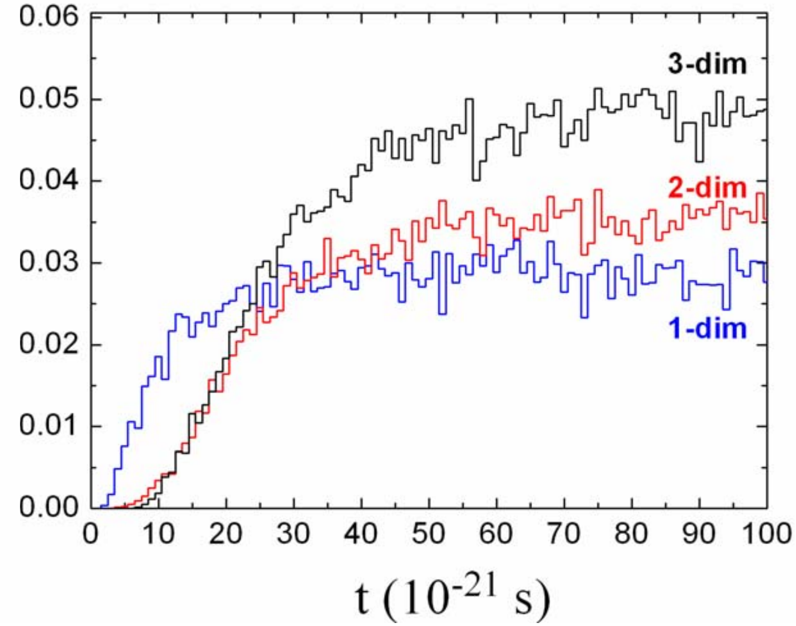
The lowest transient time – one-dimensional calculations

The results for the ^{248}Cf for E*=30MeV

two-body



one-body



The collective energy at saddle point configurations averaged over Langevin trajectories :

two-body

1d: <Ecoll> = 1.43 MeV

2d: <Ecoll> = 2.58 MeV

3d: <Ecoll> = 3.60 MeV

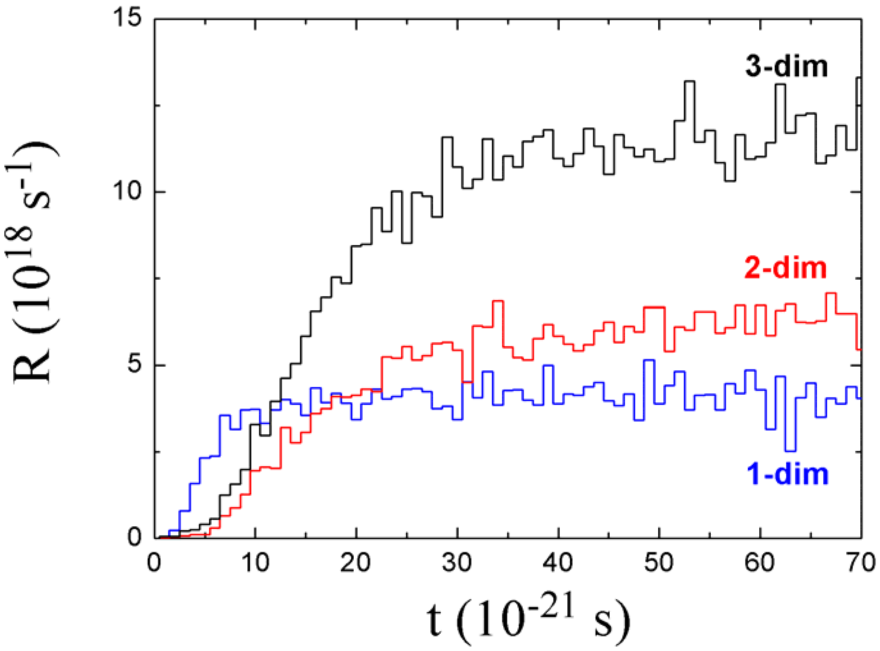
one-body

1d: <Ecoll> = 1.26 MeV

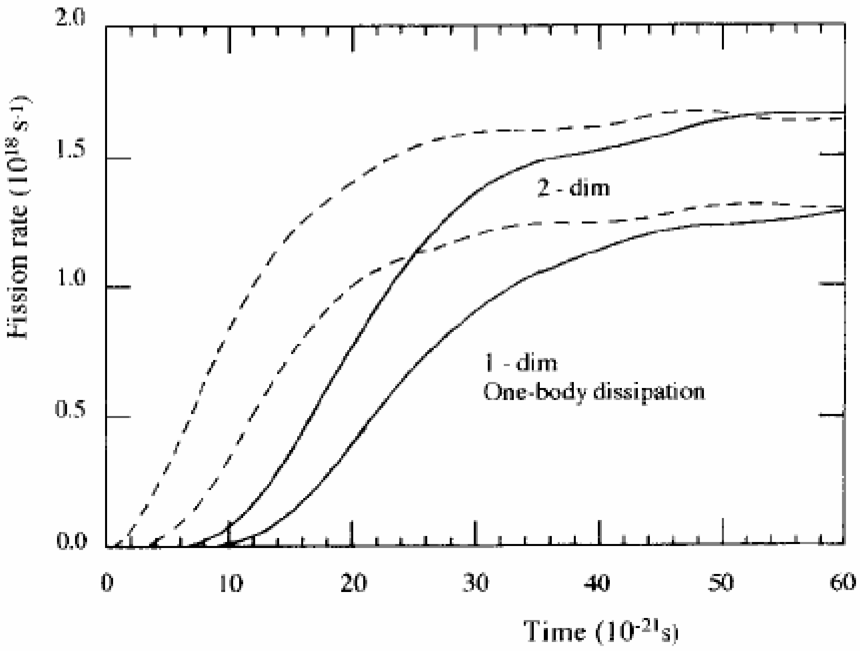
2d: <Ecoll> = 2.07 MeV

3d: <Ecoll> = 2.65 MeV

The results for the ^{213}At (one-body)



present calc.



Abe et. al. Phys. Rep. (1996)

1-dimensional versus 2-dimensional:

The same qualitative results for the stationary values, but different behavior for transient times.

Summary

- The results of R(t) calculations appreciable dependent on the number of collective coordinates involved in Langevin calculations.

- The transient time and stationary value of fission rate is larger in multi-dimensional calculations than in one-dimensional calculations.

- How the introduction of another collective coordinates will influence on the R(t)?